# Schedule

| Friday | |
|---|---|
| 9:00–9:15 | Opening |
| 9:15–10:00 | G.Fusco |
| 10:05–10:50 | M.Amar |
| 10:50–11:25 | Coffee Break |
| 11:25–12:10 | M.Freguglia |
| 12:15–13:00 | R.Scala |
| 13:00–14:30 | Lunch |
| 14:30–15:15 | A.Chambolle |
| 15:20–16:05 | N.Picenni |
| 16:05–16:40 | Coffee Break |
| 16:40–17:25 | G.Alberti |
| 20:00 | Social Dinner |

| Saturday | |
|---|---|
| 9:30–10:15 | M.Pozzetta |
| 10:20–11:05 | M.S.Gelli |
| 11:05–11:45 | Coffee Break |
| 11:50–12:30 | G.Mantegazza |
| 12:30–14:00 | Lunch |
| 14:00–14:45 | L.Bertini |
| 14:50–15:35 | M.Paolini |

# Booklet of abstracts

## Invited speakers

**Giovanni Alberti**

*Università di Pisa*

### Grain boundaries in perimeter minimizing partitions with many cells

Given a domain $E$ in the plane, I consider a minimal $N$-partition of $E$, that is, a partition that minimizes the total perimeter among all partitions of $E$ made of $N$ cells with equal area. T.C. Hales proved in 2001 that if $E$ is a flat 2-dimensional torus, then the only minimal N-partition is the regular hexagonal one (assuming that such partition exists).

But what happens for if $E$ that does not admit a regular hexagonal partition? One can show that, as expected, cells look more and more hexagonal as $N$ tends to infinity, and thus the next question concerns the "regularity" of such almost hexagonal partition: is it rigid, in the sense that the orientation of the cells is (essentially) the same through the domain? I will present evidence that the answer is negative, and that the domain splits in large blocks (grains) where the orientation of the cells is essentially constant, separated by comparatively thin regions (grain boundaries) containing many non hexagonal cells. The similarity with grain structures in the theory of dislocation in continuum mechanics is not accidental.

This is an ongoing research project with Marco Caroccia (Politecnico di Milano), Giacomo Del Nin (MPI Leipzig), Adriana Garroni and Emanuele Spadaro (Roma Sapienza).

**Micol Amar**

*Sapienza Università di Roma*

### Heat diffusion and electrical conduction in composites with imperfect contact conditions

The purpose of this talk is to present some models with imperfect interfaces which origin in the description of composites made by a hosting medium containing a periodic array of inclusions of size $\varepsilon$, where the inclusions are coated by a thin layer consisting of two sublayers of different materials (with thickness of the order $\varepsilon\eta$ and $\varepsilon\delta$, respectively), disposed in such a way that one of them is encapsulated into the other. This two-phase coating material is such that one of the two components has a low diffusivity in the orthogonal direction and the other one has a high diffusivity in the tangential direction. All the parameters $\varepsilon$, $\delta$ and $\eta$ are supposed to be very small, but with different orders. In particular, the smallness of $\delta$ and $\eta$, with respect to $\varepsilon$, leads us to perform, for fixed $\varepsilon$, a two-step concentration procedure, which produces on the resulting interface between the hosting material and the inclusions some conditions involving the jump and the mean value of the two bulk potentials and

the jump of their fluxes. Moreover, also the appearance of a new surface heat potential can happen. The concentrated problems thus obtained are then homogenized, i.e. we let $\varepsilon$ tend to zero, and we briefly discuss the different resulting models.

**Lorenzo Bertini**

*Sapienza Università di Roma*

### Large deviations with respect to motion by curvature

Consider the Allen-Cahn equation. Under diffusive rescaling of space and time, for suitably prepared initial data the limiting dynamics is described by the motion by mean curvature of the interface between the two stable phases. We consider, in space dimension $d \leq 3$, a stochastic perturbation of the Allen-Cahn equation and analyze its asymptotic in such sharp interface limit proving the large deviations upper bound. The corresponding rate function is finite only when there exists a $d-1$ interface between the two stable phases and can be written as the sum of two terms: the first takes into account "wrong" motions of the interface while the second is due to the possible occurrence of nucleation events. In particular, the zero level set of this rate function is given by the evolution by mean curvature in the Brakke formulation. Our results relies on previous analysis on the variational convergence of the action functional associated to the Allen-Cahn equation.

**Antonin Chambolle**

*Université Paris-Dauphine*

### Discrete-to-continuum crystalline curvature flow

We consider a discrete Almgren-Taylor-Wang implicit scheme, on a finite cubic grid (in any dimension) with a scale parameter h. The discrete surface tension involves only finite interactions, so that the limiting surface tension is the support function of a zonotope. We use the natural mobility defined by its polar.

We show that if the spatial and time scale are identical, and with an appropriate definition of the distance to the discrete moving set, we can recover in the limit the crystalline curvature flow corresponding to the limiting surface tension.

**Mattia Freguglia**

*Scuola Normale Superiore Pisa*

### Diffuse approximation of the Willmore functional and a conjecture of De Giorgi

We will discuss a conjecture of De Giorgi, dating back to 1991, where he proposed a possible approximation, in the sense of Gamma-convergence, of the Willmore functional based on the first variation of the Modica-Mortola functionals.

Since then several authors have investigated this problem and some modifications of the approximating functionals have been considered. For some of these modifications it has been proven that they actually approximate the Willmore functional on smooth sets.

After reviewing some of the most relevant contributions originated from this conjecture over time we will present a recent result obtained in collaboration with G. Bellettini and N. Picenni where we provide a negative answer to the original conjecture.

If time permits we will also discuss some other properties of the original functionals introduced by De Giorgi.

**Giorgio Fusco**

*Università Dell'Aquila*

### Connectivity of the phases and fine structure of minimizers in the theory of phase transitions

We study minimizers of the Allen Cahn energy with a phase transition potential, a nonnegative potential that vanishes only on a finite set of points that model the phases. We work on two dimensional domains with Dirichlet boundary conditions.

We show that the assumption of connectivity of the phases allows for a detailed description of the fine structure of minimizers. In particular one can characterize the shape and the size of the domain regions where a minimizers remains in a neighborhood of one or another of the zeros of the potential and also how these regions depend on the surface tensions.

**Maria Stella Gelli**

*Università di Pisa*

### Variational analysis of non local energies on periodically perforated domains

In the last years non local functionals have attracted great interest in view of various applications to different physical models. In this talk I will focus on non local energies of convolution-type. This kind of energies date back to the work by Bourgain, Brezis and Mironescu on fractional Sobolev norms and have been recently widely studied as continuous energies depending on finite differences appear in a natural way in the study of models of inhomogeneous media with an underlying periodic microstructure. In this respect, I will present some recent results on the asymptotics of such energies accounting for a Dirichlet type condition imposed on a periodic perforation of the domain. More in detail, I will show that the interplay among the periodicity, the size of the perforations, and the approximation parameter affects deeply the asymptotic behaviour of the energies, highlighting different phenomena

**Carlo Mantegazza**

*Università di Napoli Federico II*

### Stability for the surface diffusion flow

We study the surface diffusion flow in the flat torus, that is, smooth hypersurfaces moving with the outer normal velocity given by the Laplacian of their mean curvature. This model describes the evolution in time of interfaces between solid phases of a system, driven by the surface diffusion of atoms under the action of a chemical potential. We show that if the initial set is sufficiently "close" to

a strictly stable critical set for the Area functional under a volume constraint, then the flow actually exists for all times and asymptotically converges to a "translated" of the critical set. This generalizes the analogous result in dimension three, by Acerbi, Fusco, Julin and Morini.

Joint work with Antonia Diana e Nicola Fusco.

**Maurizio Paolini**

*Università Cattolica del Sacro Cuore - Brescia*

### Soap films and partial wetting

In the context of soap films spanning a given 1D frame, an interesting feature, unespectedly physically and mathematically feasible, show that there exist nontrivial minimal surfaces (possibly comprising singularities of the types listed by J. Taylor) that touches the wire frame only partially.

We show the results of some numerical simulations that should help in understanding the structure and shape of the surface in the vicinity of a point at the boundary of the "wetted" portion of the wire frame.

This work was done in collaboration with G. Bellettini

**Nicola Picenni**

*Scuola Normale Superiore Pisa*

### From the regularization of the Perona-Malik functional to a challenging free-discontinuity problem

We consider the Perona-Malik functional, that is an integral functional with nonconvex Lagrangian, whose formal gradient flow is the celebrated forward-backward equation introduced by Perona and Malik in the 90s. We regularize this functional either by space-discretization or by adding a higher order term, and we discuss the asymptotic behavior of minimizers when the regularization parameter tends to zero. When the space dimension is equal to one, we can show that minimizers of the regularized functionals develop a microstructure that looks like a piecewise constant function at a suitable scale, and more precisely that the blow-ups at a suitable scale of any sequence of minimizers converge to a local minimizer of a free-discontinuity problem. In this one-dimensional case, such local minimizers can be easily characterized, and turn out to be staircase-like functions. After a brief discussion of these one-dimensional results, we describe the higher-dimensional version of the limiting free-discontinuity problem, which turns out to be much more challenging. In particular, in the planar case, we show that there exists at least one symmetry-breaking local minimizer. The talk is based on joint works with Massimo Gobbino.

**Marco Pozzetta**

*Università di Napoli Federico II*

### Uniqueness of blowups for the motion by curvature of networks

A network is a finite union of embedded open curves in the plane such that their endpoints are joined at triple junctions. A motion by curvature is a one-parameter family of networks evolving so that the normal driving velocity at any point is given by the curvature vector. Hence, a motion by curvature corresponds to the $L^2$-gradient flow of the length functional on networks.

Like higher dimensional mean curvature flows, the motion by curvature of networks may develop singularities, whose behavior can be understood by studying blowups at points where the singularity occurs. A blowup is a flow obtained as the limit of a sequence of dilations centered at the singularity point.

In this talk we discuss a result that proves uniqueness of blowups for the motion by curvature of networks, that is the fact that the blowup at a singularity does not depend on the chosen sequence of rescaling factors. The proof is based on the application of a Łojasiewicz–Simon gradient inequality.

The talk is based on a work in collaboration with C. Mantegazza and A. Pluda.

**Riccardo Scala**

*Università di Siena*

### Recent developments in the relaxation of area functional

In the recent years I collaborated with Giovanni Bellettini on the project of trying to understand some properties of the relaxed area functional in dimension and codimension greater than 1. I will summarize some results obtained in the last, say, 5 years, and will discuss some recent developments. Precisely, I will introduce a notion of weak Jacobian determinant for BV maps, and show how it pops up in the analysis of the relaxed area functional of $R^2$ valued maps from a 2-dimensional domain. I will also present some perspectives and open problems.